which is the desired condition, containing one arbitrary constant and one arbitrary function, and reducing to Lamé's condition when $k = 0$.

It is easy to show that (12) is satisfied by a set of parallel planes, concentric spheres, or concentric circular cylinders, all these being parallel surfaces. It is not satisfied, as is the case with Laplace's equation, by a set of confocal ellipsoids. It was owing to the impossibility of finding other sets of surfaces to test the result, together with compunctions about Huygens' principle, that caused the writer to hold back the result. Recently, however, Somiliana has published a proof that these are the only sets of parallel surfaces that allow propagation according to Huygens' principle (*Atti Torino*, 54, (974–979). Sulle relaziono fra il principio di Huygens e l'ottica geometrica) in which he is obliged to use the formulae of differential geometry. I therefore venture to publish the above more simple result.

THE ABSORPTION OF X-RAYS BY CHEMICAL ELEMENTS OF HIGH ATOMIC NUMBERS

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Introduction.—A critical absorption wave-length characteristic of a chemical element is a wave-length such that the element absorbs X-rays longer than the critical value less than it does X-rays shorter than that value. Each chemical element has one critical absorption wave-length associated with its K series of characteristic emission lines. Duane and Hu[1] have shown that in the K series of rhodium the critical absorption wave-length is about one-fourth of one per cent shorter than that of the shortest line (the γ line) in the K emission series. The authors[2] found that the critical absorption in the K series of tungsten has a wave-length of about one-half of one per cent shorter than that of the $K\gamma$ emission line. Since the other characteristic X-ray series have longer wave-lengths than those of the K series, the K critical absorption wave-length is the shortest X-ray wave-length now known to be characteristic of a chemical element.

In the research described in this note the authors have measured the critical absorption wave-lengths in the K series of most of the available chemical elements from tungsten to uranium, both inclusive. They used an ionization spectrometer, and examined spectra of the first, second and third orders. In 1918 Duane and Shimizu[3] measured four of these wave-lengths in spectra of the first order by the ionization method. Measurements with the same spectrometer of the K critical absorption had previously been made for most of the chemical elements down to man-

ganese (atomic number 25).[4] De Broglie,[5] and Siegbahn and Jönsson[6] have published measurements of the K critical absorption by means of photographic spectrometers for the chemical elements of high atomic numbers included in this research. They examined spectra of the first order. Columns 3 and 4 in the table contain the values of the wave-lengths they give.

Apparatus and Method.—The general method of making the measurements does not differ essentially from that employed by Duane and Shimizu.[3] We have been able, however, to apply a somewhat higher voltage to the X-ray tube than in the earlier researches. The X-ray tubes we have used do not withstand a *constant* difference of potential greater than about 115,000 volts. To produce X-rays shorter than the K series of uranium, however, requires a voltage considerably in excess of this figure. By encasing the arms of the X-ray tube and also those of the kenotrons belonging to the high tension generating plant in oil baths we have been able to excite the tube with approximately constant voltages up to about 140,000 volts. We estimated these voltages with an electrostatic volt-meter, which we calibrated by measuring the current from the generating plant through a series of coils of manganin wire having a total resistance of 6.043,000 ohms. As a check on the voltage measurements and on the constancy of the difference of potential we determined the short wave-length limit of the general X-ray spectrum and calculated the voltage by means of the quantum equation $Ve = hv$. Duane and Hunt[7] showed by experiments that this law holds for the limit of the spectrum, and their results have been verified by more recent researches.

Results of the Measurements.—On platting the currents in the ioniza-tion chamber against the angular positions of the table on the spectrom-eter that supports the crystal we obtain curves, examples of which appear in figures 1 and 2. The sharp drops, a, in these curves, represent the critical absorption due to the chemical element under investigation. A layer of matter containing this element is placed in the path of the X-rays between the X-ray tube and the spectrometer. The angular dis-tance between the mid points in corresponding drops on the two sides of the zero gives us twice the glancing angle, θ, which must be substituted in the usual formula,

$$\lambda = 2d \sin \theta$$

to calculate the critical absorption wave-length. A correction for excen-tricity, amounting to about $27''$ of arc, has to be subtracted from the val-ues of θ measured on the graphs.

One pair of curves in figure 1 represents the critical absorption of uranium. These curves are of special interest, for they correspond to the shortest characteristic X-ray (of any chemical element) that has been discovered up to the present time. The centres of the drops can be
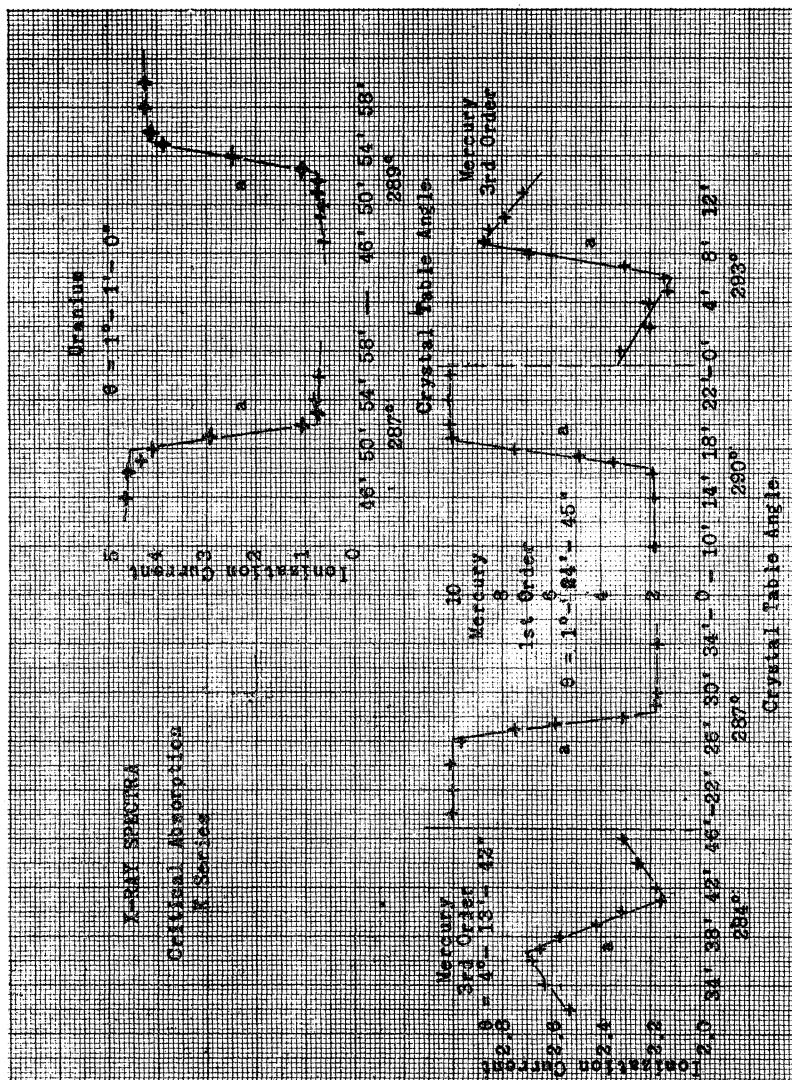
FIG. 1

estimated with reasonable certainty to within about 10″ of arc, and, as θ equals 61′ in this case, the error of precision amounts to less than one-third of one per cent.

The other curves in figure 1 and the curves in figure 2 belong to mercury and bismuth, respectively. One pair of curves represents the critical absorption of bismuth in the second order spectrum, and another pair, the critical absorption of mercury in the third order. The drops corresponding to the absorption in the second and third order spectra are very much smaller than in the first order, and for this reason it is
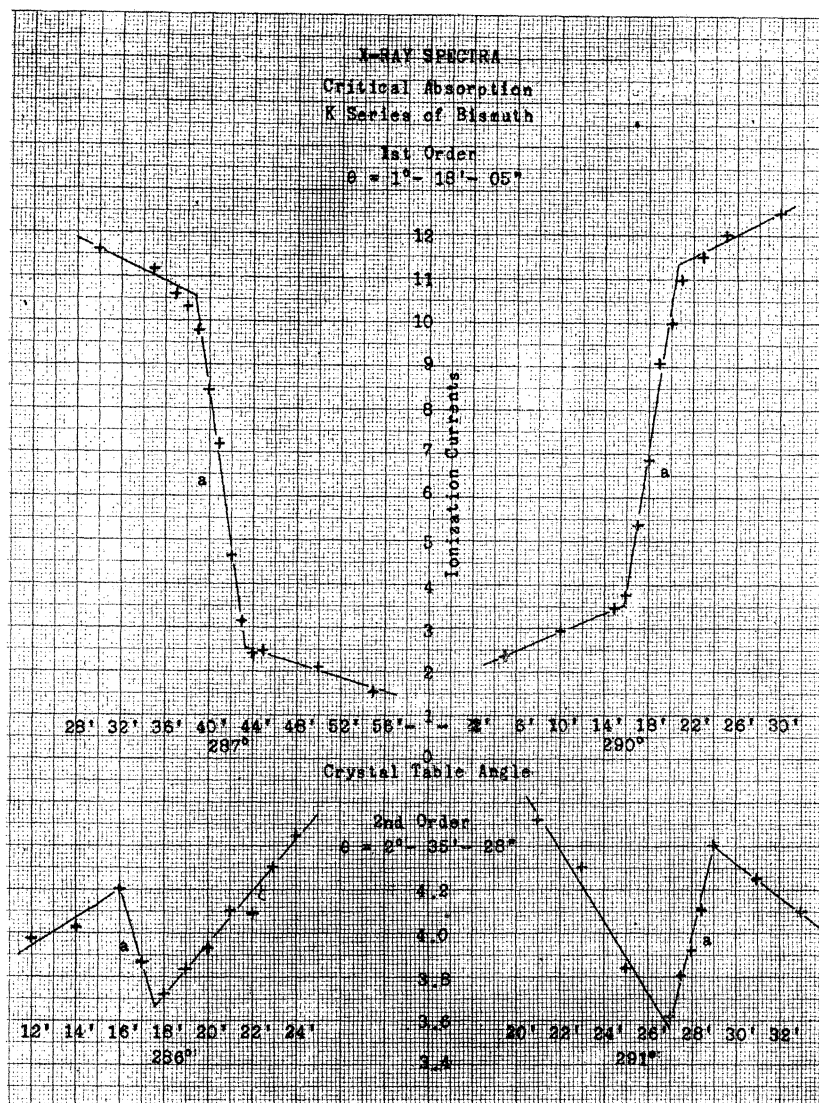
FIG. 2

more difficult to estimate their centers. The value of θ, however, is two or three times as large as in the first order.

Columns 5, 6 and 7 in table 1 contain the critical absorption wave-lengths estimated from the curves of figures 1 and 2 (and similar curves for the other chemical elements) in the spectra of the first, second and third orders respectively. Column 8 contains weighted mean values of the wave-lengths, column 9, the square roots of the wave-numbers, and

TABLE I.

K CRITICAL ABSORPTION WAVE-LENGTHS, $\lambda \times 10^8$ Cm.

For Calcite $\lambda = 6.056 \times$ s in $\theta \times 10^{-8}$ Cm.

CHEMICAL ELEMENT	ATOMIC NUMBER	DE BROGLIE	SIEGBAHN JÖNSSON	AUTHORS' VALUES					
				1ST	2ND	3RD	AVERAGE	$\sqrt{1/\lambda} \times 10^{-4}$	DIF.
Tungsten*....	74	......	......	.1783	.1781	.1780	.1781	2.370	
Platinum.....	78	.152	.1578	.1582	....	.1581	.1581	2.515	.036
Gold........	79	.149	.1524	.1537	....	.1532	.1534	2.553	.038
Mercury.....	80	.146	.1479	.1493	....	.1488	.1491	2.590	.037
Thallium......	81	.142	.1427	.1448	....	.1449	.1448	2.628	.038
Lead........	82	.138	.1385	.1412	.1409	....	.1410	2.663	.035
Bismuth.....	83	.133	.1346	.1375	.1369	....	.1372	2.700	.037
Thorium.....	90	......	.1127	.1139	.1124	....	.1131	2.974	.039
Uranium.....	92	......	.1048	.1075	......	....	.1075	3.051	.037

* The wave-lengths for tungsten were measured in coöperation with Dr. R. A. Patterson.

column 10, the difference, between these square roots calculated per unit increase in atomic number.

Discussion of the Results.—The differences tabulated in column 10 do not vary from their mean value more than one would expect from the experimental errors. This indicates that within the limits of error $\sqrt{1/\lambda}$ is a uniformly increasing function of the atomic number. The differences, however, are slightly larger than the corresponding differences between the square roots of the critical absorption wave-numbers measured in previous researches[3,4] for chemical elements of lower atomic numbers. This means that the square root of the critical absorption wave-number is not quite a linear function of the atomic number, which agrees with the results noted in the earlier papers.[3,4] The variation from the linear law, however, is too small to be clearly indicated in the range of values contained in table 1.

The wave-lengths measured in spectra of the second and third order (columns 6 and 7) appear to be, with one exception, smaller than the corresponding wave-lengths measured in the first order spectrum. The differences between the two sets of values about equal the errors of measurement. Similar differences between spectra of the various orders were observed first by Dr. Stenström[8] in his measurements of long X-rays by means of a photographic spectrometer. He attributed the effect to a slight refraction and dispersion of the X-rays by the reflecting crystal.

Our measurements in the second order spectrum for thorium were not

as satisfactory as the others. The generating plant had to be pushed to the limit of its capacity, and the fluctuations in the X-ray output were unusually large.

Our values of the wave-lengths are uniformly larger than those obtained by photographic methods. On the average these differences amount to between one and two per cent. The measurements by Siegbahn and Jönsson were made with an interesting spectrometer specially designed to eliminate errors due to the penetration of the X-rays into the reflecting crystal, etc. In an ordinary spectronometer, if the grazing angle θ, is determined by the position of the reflected beam of X-rays these errors may be either positive or negative, according to the position of the axis of rotation of the crystal with reference to what we have called the *effective* reflecting plane. Further, they may increase or decrease with the wavelength of the rays, and in limiting cases the size of the crystal would have an effect on them. Our method of using the spectrometer eliminates these errors, for we determine the grazing angle, θ, from the angle through which the crystal turns and not by the position of the reflected beam. The differences, therefore, between our values and those given by Siegbahn and Jönsson cannot be ascribed to errors of this kind. Possibly they may be due to differences in the manner of interpreting the experimental observations. If, for instance, measurements on a photographic plate were made from the point where the plate *begins* to get dark, the corresponding value of the critical absorption wave-length would be shorter than that given by our method, for we measure between points half way up the steep drops. The point where the photographic plate begins to darken corresponds to the bottom of a drop on our curves.

The wave-lengths contained in table 1 complete the series of measurements of the K critical absorption of the chemical elements that we have been making in our X-ray laboratory during the past few years.[3,4] We now have values of the K critical absorption wave-lengths for most of the chemical elements from manganese (atomic number 25) to uranium (atomic number 92) both inclusive. These wave-lengths have been measured by means of the same ionization spectrometer and with the same calcite crystal, and are therefore comparable with each other.

[1] Duane and Hu, *Physic. Rev., Ithaca, N. Y.*, Oct., 1918 (369).

[2] Duane and Stenström, *Washington, Nat. Acad. Proc.*, Aug., 1920.

[3] Duane and Shimizu, *Physic. Rev.*, Dec., 1919 (522).

[4] Blake and Duane, *Ibid.*, Dec., 1917, 697; and Duane and Hu, *Ibid.*, Dec., 1919 (516).

[5] De Broglie, *J. Physique, Paris*, May–June, 1916 (161).

[6] Siegbahn and Jönsson, *Physik. Zeits.*, 1919 (251).

[7] Duane and Hunt, *Physic. Rev.*, Aug., 1915 (166).

[8] Stenström, Doctor's *Dissertation*, Lund., 1919.